Heuristic Optimization Strategies in Finance - An Overview

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COMISEF Fellows’ Workshop: Numerical Methods and Optimization in Finance

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1 Introduction
- Financial World
- Complexity

2 Heuristic Optimization Techniques
- Classical concept
- Differential Evolution

3 Financial Applications
- Optimization heuristic strategies in finance

4 Conclusion
- Have in mind
1. **Introduction**
   - Financial World
   - Complexity

2. **Heuristic Optimization Techniques**
   - Classical concept
   - Differential Evolution

3. **Financial Applications**
   - Optimization heuristic strategies in finance

4. **Conclusion**
   - Have in mind
Financial world

Financial problems

Optimization
Complexity

Least median of squares residuals as a function of $\alpha$ and $\beta$

Multiple local minima (optima)

Apply optimization heuristic techniques
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Local search procedure

1: Generate initial solution $x^c$
2: while stopping criteria not met do
3: Select $x^n \in \mathcal{N}(x^c)$ (neighbor to current solution)
4: if $f(x^n) < f(x^c)$ then
5: $x^c = x^n$
6: end if
7: end while
Introduction

Heuristic Optimization Techniques

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Classical concept

Heuristic Techniques

Construction methods

Local search methods

Trajectory methods

Population search

Threshold accepting

Differential evolution

Tabu search

Genetic algorithms

Hybrid Meta Heuristics
Introduction

Heuristic Optimization Techniques

Classical concept

- Construction methods
- Heuristic Techniques
- Local search methods

- Trajectory methods
- Population search

- Differential evolution
- Genetic algorithms
- Hybrid Meta Heuristics
Differential Evolution

Population based heuristic with remarkable performance in continuous numerical problems.

Example


\[ r_{i,t} - r_t^s = \alpha + \beta(r_{m,t} - r_t^s) + \varepsilon_{i,t} \]  

- Least Median of Squares Estimators (LMS) (Rousseeuw and Leroy (1987)):

\[ \min_{\alpha,\beta}(\text{med}(\varepsilon_{i,t}^2)) \]
Differential Evolution

Optimal parameter estimation of CAPM using LMS estimators

1. Generate random set values $\alpha$ and $\beta$ (initial solutions)
2. Evaluate initial solutions minimizing LMS
3. Generate new candidate solutions from the initial one
4. Evaluate new candidate solutions minimizing LMS
5. Repeat until a very good solution is found
Differential Evolution

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Initialization

1. Step 1 & 2: Generate and evaluate random set values $\alpha[-1, 2]$ and $\beta[0, 1]$ (initial solutions)

2. $P(0) = \begin{pmatrix}
1 \\
2 \\
\vdots \\
d
\end{pmatrix}^{np}$

3. $\min_{\alpha, \beta}(\text{med}(\varepsilon_{i,t}^2))$
Differential Evolution

Optimal parameter estimation of CAPM using LMS estimators

Initialization

- Step 1 & 2: Generate and evaluate random set values \(\alpha[-1, 2]\) and \(\beta[0, 1]\) (initial solutions)

\[
P^{(0)} = \begin{pmatrix} 1 & \cdots & n_p \\ 2 & \cdots \\ \vdots \\ d \end{pmatrix}
\]

- \(\min_{\alpha, \beta} (\text{med}(\varepsilon^2_{i,t}))\)
Differential Evolution

Optimal parameter estimation of CAPM using LMS estimators

Generate new candidate solutions from the initial one

- Step 3a: Differential mutation

\[
P_{1,i}^{(0)} = \begin{pmatrix} 1 & 2 & \ldots & r_3 & \ldots & r_1 & \ldots & r_2 & \ldots & n_p \end{pmatrix}
\]

- \[P_{1,i}^{(v)} = P_{1,i}^{(0)} + F \times (P_{1,i}^{(0)} - P_{1,i}^{(0)})\]

- \(F\) scale factor \(\in [0, 1+]\) determines speed of shrinkage
Differential Evolution

Random crossover

Step 3b: Crossover elements from $P_{1,i}^{(0)}$ and $P_{1,i}^{(υ)}$

1. Generate for each parameter a uniform random number, $u_d \in [0, 1]$
2. Determine the crossover probability, $CR \in [0, 1]$
3. Only if $u < CR$, $P_{1,i}^{(u)} = P_{1,i}^{(υ)}$
4. else $P_{1,i}^{(u)} = P_{1,i}^{(0)}$
Differential Evolution

Optimal parameter estimation of CAPM using LMS estimators

Generate new candidate solutions from the initial one

- Step 3a: Differential mutation

\[
P_{1,i}^{(0)} = \begin{pmatrix} 1 & 2 & \cdots & r_3 & \cdots & r_1 & \cdots & r_2 & \cdots & n_p \end{pmatrix}
\]

- \( P_{2,i}^{(v)} = P_{2,r_1}^{(0)} + F \times (P_{2,r_2}^{(0)} - P_{2,r_3}^{(0)}) \)

- F scale factor \( \in [0, 1+] \) determines speed of shrinkage
Random crossover

Step 3b: Crossover elements from $P_{d,i}^{(0)}$ and $P_{d,i}^{(v)}$

1. Generate for each parameter a uniform random number, $u_d \in [0, 1]$
2. Determine the crossover probability, $CR \in [0, 1]$
3. Only if $u < CR$, $P_{d,i}^{(u)} = P_{d,i}^{(v)}$
4. else $P_{d,i}^{(u)} = P_{d,i}^{(0)}$
Differential Evolution

Optimal parameter estimation of CAPM using LMS estimators

Initialization

- Step 4: Evaluate new candidate solutions minimizing LMS
- if $f(P_{u,i}^{(u)}) < f(P_{i}^{(0)})$ then $f(P_{i}^{(0)}) = f(P_{i}^{(u)})$
Differential Evolution

Optimal parameter estimation of CAPM using LMS estimators

Initialization

- Step 4: Evaluate new candidate solutions minimizing LMS
- if \( f(P_{i}^{(u)}) < f(P_{i}^{(0)}) \) then \( f(P_{i}^{(0)}) = f(P_{i}^{(u)}) \)
Differential Evolution

Optimal parameter estimation of CAPM using LMS estimators

Initialization

- Step 5: Repeat steps 3 & 4 until a good solution is found or for a predefined number
## Differential Evolution

Optimal parameter estimation of CAPM using LMS estimators

### Initialization

- Step 5: Repeat steps 3 & 4 until a good solution is found or for a predefined number
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## Optimization heuristic strategies in finance

### Portfolio Optimization
- Transaction Costs
- Cardinality constraints
- Index tracking
- VAR

### Robust methods
- Model estimation
- Model selection

### Clustering
- Financial forecasting
- Portfolio improvement
- Mutual funds style
Optimization heuristic strategies in finance

**Portfolio Optimization**
- Transaction Costs
- Cardinality constraints
- Index tracking
- VAR

**Robust methods**
- Model estimation
- Model selection

**Heuristics**
- Financial forecasting
- Portfolio improvement
- Mutual funds style

**Clustering**
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Optimization heuristic strategies in finance

**Portfolio Selection**

Dueck and Winker (1992) applied Threshold Accepting

1. Portfolio optimization using various risk measures
2. Index tracking to mutual fund replication
3. Currency portfolio optimization
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**Portfolio Selection**

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1. Portfolio optimization using various risk measures
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Model estimation

1. Risk estimation and GARCH models
2. Indirect estimation and Agent Based Models
3. Yield curve estimation
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Yield curve estimation

Fig. 6. Error in the estimation of the interest rates. Comparison of the traditional non-linear least squares and the GA for the Nelson and Siegel (1987) on the left and for the Svensson (1994) function on the right for 1 year (top graph) and 10 years (bottom graph) government bonds.
Model selection

1. Risk factor selection (Asset Pricing Theory model (APT))

\[ r_{i,t} = \alpha + \sum_{f=1}^{k} \beta_f r_{f,t} + \varepsilon_{i,t} \] (3)

2. Selection of bankruptcy predictors

\[ BMW \ assets = 0.8 \ German \ factor + 0.2 \ US \ factor + 0.9 \ automotive \ factor + 0.1 \ finance \ factor + BMW \ non \text{-} systematic \ risk \] (4)
Model selection

1. Risk factor selection (Asset Pricing Theory model (APT))

\[ r_{i,t} = \alpha + \sum_{f=1}^{k} \beta_f r_{f,t} + \varepsilon_{i,t} \]  \hspace{1cm} (3)

2. Selection of bankruptcy predictors

\[
BMW \text{ assets} = 0.8 \text{German factor} + 0.2 \text{US factor} \\
+ 0.9 \text{automotive factor} + 0.1 \text{finance factor} \\
+ BMW \text{ non–systematic risk} \hspace{1cm} (4)
\]
Optimization heuristic strategies in finance

Clustering

1. Bankruptcy prediction
2. Credit risk rating
3. Portfolio performance improvement
4. Identify optimal number of clusters
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**Heuristic optimization techniques**
- **Flexible** to tackle many **complex** optimization problems

**Flexibility cost**
- Might be computationally more demanding than traditional methods (not a limitation nowadays)
- Results carefully interpreted
Summary

- Apply Optimization Heuristics